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Research Paper

On empirical likelihood option pricing

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ABSTRACT

The Black–Scholes model is the golden standard for pricing derivatives and options in the modern financial industry. However, this method imposes some parametric assumptions on the stochastic process, and its performance becomes doubtful when these assumptions are violated. This paper investigates the application of a nonparametric method, namely the empirical likelihood (EL) method, in the study of option pricing. A blockwise EL procedure is proposed to deal with dependence in the data. Simulation and real data studies show that this new method performs reasonably well and, more importantly, outperforms classical models developed to account for jumps and stochastic volatility, thanks to the fact that nonparametric methods capture information about higher-order moments.

Keywords: nonparametric; option pricing; empirical likelihood; robust; blocking time series.

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1 INTRODUCTION

Since the seminal works of Black and Scholes (1973) and Merton (1973), option-valuation methodologies have developed extensively. The Black–Scholes model has become one of the most well-known discoveries in the finance literature, relating the cross-sectional properties of option prices with the underlying assets' returns distributions. However, Rubinstein (1985) and Melino and Turnbull (1990) point out several limitations in the Black–Scholes model due to strong assumptions, such as the nonnormality of returns, stochastic volatility (implied volatility smile), jumps and others. Both parametric and nonparametric approaches have been proposed to deal with these issues.

Scott (1987), Hull and White (1987) and Wiggins (1987) extend the Black–Scholes model and allow the volatility to be stochastic. Heston (1993) develops a closed-form solution for option pricing when the underlying asset's volatility is stochastic. Duan (1995) proposes a generalized autoregressive conditional heteroscedasticity (GARCH) option pricing model in an attempt to explain some systematic biases associated with the Black–Scholes model. Heston and Nandi (2000) provide a closed-form solution for option pricing, with the underlying asset's volatility following a GARCH(p, q) process. Bates (1996) and Bakshi *et al* (1997) derive an option pricing model with stochastic volatility and jumps. Kou (2002) provides a solution to pricing the option with the double exponential jumps diffusion process. Carr and Madan (1999) introduce the fast Fourier transform approach to option pricing, given a specified characteristic function of the return, which provides an efficient computational algorithm to calculate the option prices (for more information, see, for example, Duffie *et al* (2000), Bakshi and Madan (2000) and Carr and Madan (2009)). All of these methods assume a parametric form of either the distribution of the underlying asset returns or the characteristic function of the underlying asset returns.

Nonparametric approaches have also been proposed to capture the underlying asset and option price data in order to reconstruct the structure of the diffusion process. For example, Hutchinson *et al* (1994) apply neural network techniques to price derivatives. Ait-Sahalia and Lo (1998) use the kernel regression to fit the state-price density implicitly in option pricing. Ait-Sahalia (1996) proposes a nonparametric pricing estimation procedure for interest rate derivative securities under the assumption that the unknown volatility is independent of time. Stutzer (1996) adopts the canonical valuation method, which incorporates the nonarbitrary principle embodied in the formula for calculating the expectation of the discounted value of assets under the risk-neutral probability distribution.

One of the most important nonparametric methodologies is the empirical likelihood (EL), which conducts likelihood-based statistical inference by profiling a nonparametric likelihood (see, for example, Owen 1988, 1990, 2001; DiCiccio and Romano 1989;

Hall and La Scala 1990). For the application of the EL method to time series, see Mykland (1995), Chuang and Chan (2002) and Ling and Chan (2006), among others. Kitamura (1997) introduces a blockwise EL method for a weakly dependent time series. Nordman *et al* (2007) modify the blockwise methods to cope with various dependence structures and ultimately achieve better finite sample performance. Yau (2012) studies the application of EL to long-memory time series.

In this paper, we implement the EL method to price derivatives or options under risk-neutral measures. First, we construct an empirical probability constraint using historical holding period return time series observations without assuming the distribution family of the returns. Further, we view the derivative/option price directly as the parameter of interest in the EL optimization procedure. An EL-based estimate of the parameter (eg, call option price) is obtained, and the asymptotic properties of the EL ratio are studied. We further introduce a blockwise EL procedure for weakly dependent processes. Monte Carlo simulation and the empirical results for Standard & Poor's 500 (S&P 500) index options are discussed.

The remainder of this paper is organized as follows. Section 2 provides a detailed EL procedure in option pricing. Asymptotic properties are discussed and a robust confidence interval is constructed. Section 3 demonstrates the empirical performance of EL option pricing, including both Monte Carlo simulation and S&P 500 index options. Section 4 concludes the paper with discussions.

2 EMPIRICAL LIKELIHOOD IN OPTION PRICING

Let $P(t)$ be the underlying asset price at time t; let $D(t)$ be the future dividend at time t; let $r(s,t)$ be the gross risk-free interest rate during time s and t, with $r(t,t) = 1$; let $\mathcal{P}$ be the physical probability measure; and let $\mathcal{Q}$ be the risk-neutral probability measure (see Huang and Litzenberger 1988), under which the price process plus the accumulated dividends are martingales after normalization if no arbitrage exists in the pricing systems. To be specific, the latter leads to the following pricing formula:

$$\begin{aligned} P(t) &= E^{\mathcal{Q}}\biggl[\frac{P(T) + \sum_{s=t}^{T} D(s)r(s,T)}{r(t,T)}\biggr] \\ &= E^{\mathcal{P}}\biggl[\frac{P(T) + \sum_{s=t}^{T} D(s)r(s,T)}{r(t,T)}\frac{\mathrm{d}\mathcal{Q}}{\mathrm{d}\mathcal{P}}\biggr]. \end{aligned} \tag{2.1}$$

Here, $\mathrm{d}\mathcal{Q}/\mathrm{d}\mathcal{P}$ is the Radon–Nykodym density of the marginal measure. One can price an option or a derivative security by evaluating the expected discounted value under $\mathcal{Q}$. For example, the call option price with strike price K and expiry date T is given by

$$C(t,T) = \frac{E^{\mathcal{Q}}\max[P_T - K, 0]}{r(t,T)}. \tag{2.2}$$

The following subsection illustrates the idea of estimating $C(t,T)$ through EL coupled with the change-of-measure constraint.

2.1 The estimating procedure

Suppose historical data is available in the format

$$\{(P(t), D(t)),\ t = -1, -2, \dots, -H\}.$$

A nonparametric way of estimating the option price could be built on approximating $\mathcal{Q}$ by a discrete distribution supported on the observed value of the option price; namely, $\text{HPR}(-i-T,-i)/r(-i-T,-i)$, $1 \leqslant i \leqslant H-T$, with the corresponding probability denoted by π_i. Here, $\text{HPR}(s,t)$ is the holding period return between times s and t. If there is no dividend, $\text{HPR}(-i-T,i) = P(-i)/P(-i-T)$. Then, (2.1) can be approximated by

$$1 = \sum_{i=1}^{H-T} \frac{\text{HPR}(-i-T,-i)}{r(-i-T,-i)}\pi_i. \tag{2.3}$$

Correspondingly, we can estimate the option price by approximating (2.2) by

$$\hat{C}(t,T) = \sum_i \frac{\max[P_i(T)-K,0]}{r(t,T)}\pi_i. \tag{2.4}$$

Note that the choice of π_i subject to (2.3) is not unique. Stutzer (1996) uses the idea of maximum entropy, namely maximizing $\sum_{i=1}^{H-T} \pi_i \log \pi_i$ subject to (2.3). Here, we adopt the EL method (Owen 1988) by changing the objective function from entropy to EL, namely maximizing $\sum_{i=1}^{H-T} \log \pi_i$. This objective function can be easily interpreted as a nonparametric loglikelihood function; hence, the whole optimization procedure in our method can be interpreted as a maximum likelihood method, which is considered more efficient than a maximum entropy method. Moreover, Baggerly (1998) proposes a general class of EL-type methods, which contains both $\sum \log \pi_i$ and $\sum \pi_i \log \pi_i$ as special cases. In addition, Baggerly (1998) proves the EL used in this paper is the only method in the general class that has a higher-order correction of the large sample properties. We refer to Kitamura (1997) for a more detailed form of the higher-order correction. Meanwhile, noting that the sequence $\text{HPR}(-i-T,-i)/r(-i-T,-i)$, $1 \leqslant i \leqslant H-T$, possesses a reasonable amount of dependence, we suggest adopting the blockwise version of the algorithm as follows. Group the data into Q blocks, where length M is the length of the moving block. Set L to be the step size of the moving block. We obtain block weight π_i^* by maximizing $\sum_{i=1}^{H-T} \log \pi_i^*$ subject to

$$1 = \sum_{i=1}^{Q} \pi_i^* \left[\frac{1}{M}\sum_{j=1}^{M} \frac{\text{HPR}(-i*L-j-T,-i*L-j)}{r(-i*L-j-T,-i*L-j)}\right]. \tag{2.5}$$

Then, estimate the option price by

$$C = \sum_{i=1}^{Q} \left[\frac{1}{M} \sum_{j=1}^{M} \frac{\max[P_{i*L-j}(T) - K, 0]}{r(-i * L - j - T, -i * L - j)} \right] \pi_i^*. \tag{2.6}$$

This blocking idea is studied by Kitamura (1997), who argues that using blockwise methods offers a much better empirical performance for weakly dependent processes in moving-average noise terms. The estimation procedure in the spirit of Kitamura (1997) is slightly different,

$$\max_{C, \pi_i^*} \sum_{i=1}^{Q} \log \pi_i^*, \tag{2.7}$$

subject to constraints (2.5), (2.6) and

$$\sum_{i=1}^{Q} \pi_i^* = 1,$$

$$\pi_i^* > 0,$$

and the maximizing C is our estimator. The estimated risk-neutral measure weights pi_i^* have the following form:

$$\pi_i^* = \left\{ Q \left(1 + \gamma \left[\frac{1}{M} \sum_{j=1}^{M} \frac{\mathrm{HPR}(-i * L - j - T, -i * L - j)}{r(-i * L - j - T, -i * L - j)} - 1 \right] \right) \right\}^{-1},$$

where γ is a Lagrange multiplier. These weights are similar to the Gibbs canonical probability in Stutzer (1996), because they put small weights when the rates of return of underlyings are far from risk-free returns. In addition, Peng (2015) shows that these two approaches yield the same asymptotic property. In our simulation below, we adopt the second method, since it is well known and there is an existing package for implementation. In particular, Qin and Lawless (1994) provide a Lagrangian with multipliers approach to solve the above-mentioned optimization problem. We can either apply the numerical optimization process or derive the solution, similar to Qin and Lawless (1994). For more details about the Lagrangian optimization or the basic properties of the EL procedure, see Owen (1990) and Qin and Lawless (1994).

2.2 Asymptotic properties

In this subsection, we discuss some basic asymptotic properties of the option price with respect to the EL process ((2.6) and (2.7)), which helps us to understand the asymptotic distribution of our estimate and conduct further inference.

THEOREM 2.1 *Consider that*

$$f(\mathrm{HPR}_t, C) = \left(\frac{\max[P_i(T) - K, 0]}{r(t, T)} - C, \frac{\mathrm{HPR}(-t - T, t)}{r(t - T, t)} - 1\right)^T,$$

and further assume that

(i) *the derivative price* (C) *is in a compact set* Θ*;*

(ii) C_0 *is a unique solution of* $E(f(\mathrm{HPR}_t, C)) = 0$*;*

(iii) *for sufficiently small* $\delta > 0$ *and* $\eta > 0$*,*

$$E\Big[\sup_{C^* \in O(C,\delta)} \|f(\mathrm{HPR}, C^*)\|\Big] < \infty$$

for all $C \in \Theta$*;*

(iv) *if a sequence of* C_j, $j = 1, 2, \dots$, *converges to some* C *as* $j \to \infty$, $f(\mathrm{HPR}_t, C_j)$ *converges to* $f(\mathrm{HPR}_t, C)$ *for all* HPR_t *except on a null set, which may vary with* C*;*

(v) C_0 *is an interior point of* Θ*;*

(vi) $\mathrm{Var}(H^{-1/2} \sum_{i=1}^{H} f(\mathrm{HPR}_i, C_0)) \to S > 0$*; and*

(vii) *for a blockwise EL approach, we further assume the weak dependent condition* $\sum_{k=1}^{\infty} \alpha_X(k)^{1-1/d} < \infty$ *for some constant* $d > 1$.

We also require additional assumptions:

$$E\|f(\mathrm{HPR}_t, C_0)\|^{2d} < \infty, \quad \text{for } d > 1,$$
$$E \sup_{C^* \in O(C_0,\delta)} \|f(\mathrm{HPR}_t, C^*)\|^{2+\varepsilon} < K, \quad \text{for some } \varepsilon > 0.$$

Then,

$$\mathrm{LR}_0 = 2\sum_{i=1}^{Q} \log(1 + \gamma(\hat{C})^T f(\mathrm{HPR}_i, \hat{C})) \to_{\text{dist}} \chi_1^2,$$

where K *is a finite number,* $\gamma(\hat{C})$ *is the Lagrange multiplier vector and* Q *is the total number of states. Particularly for the nonblockwise EL case (ie, (2.6)),* $Q = H - T$.

Theorem 1 provides an asymptotic distribution of the likelihood ratio LR_0, which can be further applied to inference of the estimate. We omit the detailed proof here.[1] For independent observations of HPR_i, we require only the assumptions (i)–(vi) to

[1] Our proof is a direct consequence of Theorems 1 and 2 in Kitamura (1997).

have the asymptotic property of the likelihood ratio; for weak-dependent observations of HPR_i, assumption (vii) is also required. Given the simple fact that the chi-squared distribution is the square of a normal distribution, the distribution of the errors measured by the likelihood ratio will be close to the white noise when sample size goes to infinity. This means that our estimator will eventually capture almost all of the information in the data.

3 EMPIRICAL RESULTS

In this section, we first compare our method with several popular option pricing models through Monte Carlo simulation, and then conduct an empirical analysis on the option pricing for the S&P 500 index call options.

3.1 Monte Carlo simulation

3.1.1 Black–Scholes model

Following Hutchinson *et al* (1994), Ait-Sahalia and Lo (1995) and Stutzer (1996), we generate a geometric Brownian motion process with a 10% drift and 20% annualized volatility. First, we simulate two years of historical daily stock returns with $253 \times 2 = 506$ observations. We repeat this for 200 samples. For each sample, three different prices are calculated:

(1) the estimated price by the EL option pricing procedure;

(2) the estimated price by the Black–Scholes model with historical volatility; and

(3) the actual price by the Black–Scholes model with actual volatility.

The performances of the first two prices are compared based on the mean absolute percentage error (MAPE) with respect to the third price, which is considered to be the true price. The comparison is made at different price-to-strike-price ratios (ie, $P/X = \frac{9}{10}, 1, \frac{9}{8}$) and different expiration dates (ie, $T = \frac{1}{13}, \frac{1}{4}, \frac{1}{2}$).

Table 1 provides the simulation performance: panel (a) reports the MAPE of the EL option price, and panel (b) reports the MAPE of the historical volatility-based Black–Scholes price (Hist Var). In a perfect Black–Scholes world, the Black–Scholes formula using historical volatility outperforms the EL option pricing methodology. This is because the Black–Scholes formula only requires second moment information, and 506 observations can provide a very good estimate of the second moment; the EL method, meanwhile, automatically captures the higher-order moment information, which is not beneficial to pricing options in a perfect Black–Scholes world.

We are also interested in the accuracy of the EL option pricing for different moneyness and days to maturity. The EL option pricing method gives a very good performance in pricing the in-the-money (ITM) options with small MAPE; however, the

TABLE 1 Monte Carlo simulation in a Black–Scholes market.

Panel (a)

	Hist Var versus ideal BS	Years to maturity 1/13	1/4	1/2
Moneyness (P/K)	9/10	0.2124	0.0987	0.0687
	1	0.0327	0.0305	0.0284
	9/8	6.375×10^{-4}	0.0047	0.0080

Panel (b)

	EL versus ideal BS	Years to maturity 1/13	1/4	1/2
Moneyness (P/K)	9/10	0.724	0.514	0.537
	1	0.088	0.149	0.230
	9/8	0.003	0.025	0.058

The mean absolute percentage error (MAPE) of the EL option price to the ideal Black–Scholes price (panel (a)), and the historical volatility-based Black–Scholes price to the ideal Black–Scholes price (panel (b)) for different combinations of the relative exercise prices (P/K) and time to expiration date. The price dynamics follow the geometric Brownian motion, with $\mu = 0.1$ and $\sigma = 0.2$. The relative exercise prices (P/K) are chosen as in Rubinstein (1985) and Stutzer (1996). The time to expiration dates are 1/13, 1/4 and 1/2 years.

MAPE is very significant for out-of-the-money (OTM) options. The at-the-money (ATM) option pricing error is in between. However, the pricing errors have different patterns for ITM, ATM and OTM options. For ITM and ATM options, the fewer days to maturity, the smaller the pricing errors. For OTM options, the fewest days to maturity case has the largest pricing error, with a possible reason being that the price magnitude of the OTM options with very few days to maturity is already very small.

3.1.2 Stochastic volatility jump model

Bates (1996) adds a compound Poisson process to the Heston stochastic volatility model to account for the rare sudden drift of some financial assets. The stochastic processes are defined as follows:

$$\begin{gathered} \mathrm{d}S/S = \mu\,\mathrm{d}t + \sqrt{V}\,\mathrm{d}Z + k\,\mathrm{d}q, \\ \mathrm{d}V = (\alpha - \beta V) + \sigma_v \sqrt{V}\,\mathrm{d}Z_v, \\ \mathrm{cov}(\mathrm{d}Z, \mathrm{d}Z_v) = \rho\,\mathrm{d}t, \\ P(\mathrm{d}q = 1) = \lambda\,\mathrm{d}t, \\ \ln(1 + k) \sim N(\log(1 + \kappa), \delta^2). \end{gathered}$$

TABLE 2 MAPE of the four methods under the Bates model.

Jump parameters	EL	Stutzer	Historical Black–Scholes	Heston model
$\kappa = -0.001$, $\delta = 0.019$	0.3680	0.3804	0.5734	2.428×10^{-7}
$\kappa = 0.1$, $\delta = 0.5$	0.3759	0.4448	0.6062	0.4765

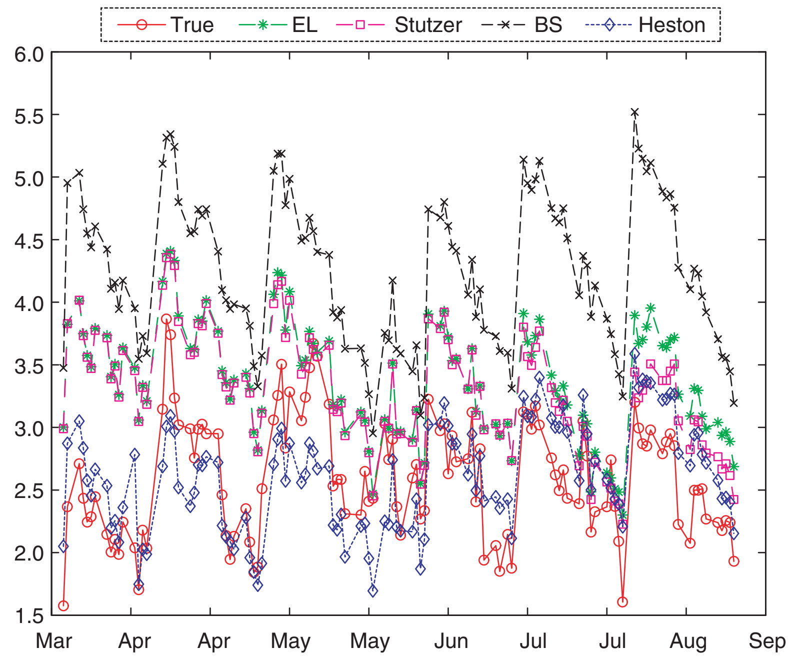
Here, we use the parameter estimates in Bates (1996) to produce simulated stock prices and European option prices. We compare our nonparametric option pricing method with that of Stutzer (1996) as well as the historical Black–Scholes and Heston models. We summarize our results in Table 2. The κ and δ are the mean and standard deviations of the sizes of jumps. From Table 2, we can see that when jump sizes are small, as is the case in the second row, our method beats the historical Black–Scholes model, but it loses to the Heston model. This is because when jump sizes are sufficiently small, the Bates model is extremely close to the Heston model, and, hence, calibration of the Heston model is more or less the same as using a parametric method with the true likelihood function. We know that the parametric likelihood method always achieves the lowest error bound when we use the right likelihood functions. When the jump sizes are large, however, as is the case in the third row of Table 2, our nonparametric method not only outperforms the other methods, but also performs consistently well, whether the jump sizes are large or small.

3.2 S&P 500 index options

We also implement the EL option pricing method in pricing S&P 500 index options. The daily return data is from the Center for Research in Security Prices (CRSP) and the option data is from OptionMetrics. The daily return data is from January 2011 to December 2012. We use daily return data from 2011 as our formation period and test its performance against daily index options pricing from 2012, comparing our results with the historical volatility-based Black–Scholes model and the true values. We only keep the options that have moneyness closest to 1 and days to maturity between 15 and 50.

Figure 1 shows the time series of the option prices. The red line is the true value of the market daily close price, the green line is the EL option price, the black line is the Black–Scholes option price using historical volatility, the blue line is the Heston stochastic volatility option pricing using least-squares calibration and the purple line is the method from Stutzer (1996). Due to the stock price movement, the true option prices vary from 1.5 to 3.7; however, the historical volatility-based Black–Scholes

FIGURE 1 Comparison of the S&P 500 index option prices and EL option prices.



The time series of three S&P 500 index option prices. We only keep the options that have moneyness closest to 1 and days to maturity between 15 to 50. The red line is the true value of the market daily close price, the green line is the EL option price, the black line is the Black–Scholes option price using historical volatility, the blue line is Heston stochastic volatility option pricing using least-squares calibration and the purple line is the method from Stutzer (1996).

option prices are consistently overpriced for the ATM call options, as is documented in Hull and White (1987). In contrast, our EL option prices are closer to the true option market prices. This is because our methodology also captures the high-order moment information in the EL procedure, while the historical volatility-based Black–Scholes option model only captures the second moment information.

4 CONCLUSION

In this paper, we introduced an EL method to price derivatives under a risk-neutral measure. Based on Monte Carlo simulations and S&P 500 index option data, we showed that our method outperforms classical alternative models (Black–Scholes, Heston and Bates), thanks to our advantage in capturing higher-order moment information.

DECLARATION OF INTEREST

The authors report no conflicts of interest. The authors alone are responsible for the content and writing of the paper.

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